A Notation System for Slide Rule Operations

Robert A. James

Imagine a chess instruction manual that stated: "Move the white pawn in front of the white King two squares. Then move the black pawn in front of the black King two squares. Then move the left white Knight two squares forward and one square to the right. Then …"

Or a contract bridge manual that stated: "South holds the following cards: the Ace, Queen and Three of Spades, … [minutes later] West, having taken the last trick, leads with the King of Diamonds, thus establishing the suit…"

Absurd, yes? We would all expect the first case to be summarized **1. e4 e5 2. Nc3** and the second case to be displayed as a matrix array of card values and suits, followed by the bids and contract and then by the playing of some tricks. Notation systems allow us quickly and succinctly to communicate complex information.

Yet consider the typical passage in a slide rule instruction manual:

> As an example, the equation $x^2 + 10x + 15 = 0$ will be used. We set the left index of CI opposite the number 15 on the D scale. We then move the hairline until the sum of CI and D scale readings, at the hairline, is equal to 10. This occurs when the hairline is set at 1.84 on D, the simultaneous reading on CI being 8.15. The sum $x_1 + x_2 = 1.84 + 8.15 = 9.99$, sufficiently close to 10 for slide rule accuracy. Roots or values of x are therefore $-x_1 = -1.84$ and $-x_2 = -8.15$. Obviously the values of x solving the equation $x^2 - 10x + 15 = 0$ will be +1.84 and + 8.15 since in this case A is negative, equal to -10.

E. I. Fiesenheiser, *Post Versalog Slide Rule Instruction Manual* (1951) pp. 20-21.

Occasionally a brave soul has attempted to develop a notation system for a more concise and clear mapping of symbols to actions to be taken or readings to be made on a slide rule of any configuration. No such system has gained general acceptance.

I have reviewed prior attempts and here propose a notation system. I then apply the system to two of my favorite examples of a famous result that can be obtained with a simple slide rule of any manufacture.

TABLE 1. Proposed Notation System

Command	Meaning
MUCO # [Scale]	Move cursor over # on [Scale]
SLIN # [Scale]	Slide Left Index to # on [Scale]
SRIN # [Scale]	Slide Right Index to # on [Scale]
SLUC # or Index [Scale]	Slide left under cursor to # or Index on [Scale]
SRUC # or Index [Scale]	Slide right under cursor to # or Index on [Scale]
RDAC [Scale] →	Read at cursor on [Scale] → #
RDLI [Scale] →	Read Left Index on [Scale] → #
RDRI [Scale] →	Read Right Index on [Scale] → #
FLIP	Flip slide rule over
[other information]	First or second decade for A/B/K, third decade for K, special hairlines on cursor, gauge marks on rule, etc.—indicate in brackets

Example 1: The Speed of Light

In 1862, James Clerk Maxwell compiled Gauss's laws for electric and magnetic charge, Faraday's law of magnetic induction, and Ampere's law of electric induction—and then, without empirical supporting evidence, added his own term to the latter to produce the Ampere-Maxwell law.

Collectively they are the famous *Maxwell equations of electromagnetism*:

- **Gauss's Law for Electricity:** $\nabla \cdot E = \rho / \varepsilon_0$
- **Gauss's Law for Magnetism:** $\nabla \cdot B = 0$
- **Faraday's Law of Induction:** $\nabla \times E = -\partial B/\partial t$
- **Ampere-Maxwell Law:** $\nabla \times B = \mu_0 J + \mu_0\varepsilon_0\, \partial E/\partial t$

(In the forms developed by Oliver Heaviside in 1881, that is.)

By making a number of simplifying assumptions, the propagation of electricity and of magnetism can be expressed as

$\partial^2E/\partial x^2 = \mu_0\varepsilon_0\, \partial^2E/\partial t^2$ and
$\partial^2B/\partial x^2 = \mu_0\varepsilon_0\, \partial^2B/\partial t^2$.

Each is a second-order partial differential equation—in fact, what physics students would recognize as a *wave equation* of the form

$\partial^2\varphi(x,t)/\partial x^2 = (1/v^2)\,(\partial^2\varphi/\partial t^2)$,

where v is the speed of the wave in question.

Thus, the speed of the electric wave and the speed of the magnetic wave are *both*

$$v = \sqrt{1/\mu_0\varepsilon_0}\,.$$

The magnetic permeability of free space (the ability of a material to align itself with a magnetic field), μ_0, is

$4\pi \times 10^{-7}$ kg m s^{-2} A^{-2} ,

or, since the derived unit coulomb C is an ampere-second, A-s,

12.56×10^{-7} kg m C^{-2} .

The electric permittivity of free space (the density of electric field lines that a material permits to form), ε_0, is about

8.85×10^{-12} C^2 kg^{-1} m^{-3} s^2.

Multiplying and cancelling the units,

$\mu_0\,\varepsilon_0 = 12.56 \times 8.85 \times 10^{-7} \times 10^{-12}$ m^{-2} s^2.
Thus, the numerical value of the speed is

$$v = \sqrt{\frac{1}{12.56 \times 8.85 \times 10^{-19}}}$$

That **can be solved using a slide rule!**

First calculate the denominator. The C and D scales of the slide rule yield

12.56 x 8.85 = 111.2 *(modern figure: 111.21)*

SRIN 885 D
MUCO 1256 C
RDAC D → 1112

Keep track of 10^{-19} m^{-2} s^2 separately. Make an even exponent by expressing it as 11.12×10^{-18}.

Next calculate the fraction, namely the reciprocal of 11.12×10^{-18} m^{-2} s^2 . Flip the unit exponents between positive and negative. The C and CI scales of the slide rule yield

0.0899×10^{18} m^2 s^{-2}. In proper scientific notation form, that's 8.99×10^{16} m^2 s^{-2}.

MUCO 1112 C
RDAC CI → 900

Finally, calculate the square root. Easy street. For the significant digits of the typical slide rule, 8.99 is indistinguishable from 9. The square root of nine is simply three. Divide the exponents and units in half.

So, the square root, either on the A and D scales or simply upon inspection, is

$$\mathbf{v = 3 \times 10^8\ m\ s^{-1}}\,.$$

MUCO 900 A [first decade]
RDAC D → 300

That's **300 million meters per second**. Sound familiar?

That's close to what Maxwell already knew to be **the speed of light in free space,** what we now define precisely as 299,792,458 meters per second.

(Using the data available at the time, Maxwell actually obtained a velocity of 310,740,000 meters per second (1.0195×10^9 ft/s).)

In his paper *A Dynamical Theory of the Electromagnetic Field,* Philosophical Transactions of the Royal Society of London, vol. 155, p. 459 (1865) (read at a meeting in 1864), Maxwell wrote:

> The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.

The same speed was no *proof* as yet that light *was* electromagnetism. After all, gravitational waves also travel at the speed of light. But the electromagnetic nature of light was soon shown. The propagation of electromagnetic radiation at many frequencies was demonstrated scientifically by Hertz in 1888 and commercially by Marconi in 1901.

Of this paper and Maxwell's related works, fellow physicist Richard Feynman said: *"From the long view of the history of mankind – seen from, say, 10,000 years from now – there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electromagnetism."*

There was much joy and rejoicing in the land. Still, some questioned the result. An absolute speed of the wave seemed to offend Galilean relativity. Shouldn't the rate depend on the motion of the reference frame? That constant speed caused nagging concern for a few more decades until a Swiss patent-office clerk came along, packing a Nestler No. 23 slide rule.

Example 2: The Age of the Shroud of Turin

The Shroud of Turin, said to be the burial linen on which the image of Jesus is miraculously imprinted, was first exhibited in Lirey, France, in 1354. Denounced even at the time as a forgery, it was acquired by the House of Savoy and displayed in Torino continuously since 1578. It is now owned by the Catholic Church, which neither endorses nor rejects its authenticity.

Interest in the object rose after 1898 when black-and-white photographs clarified the image of a face. But a 1978 study found that the image had been painted (or painted over, some believers say) with red pigment. Then in 1988 three independent laboratories in Zurich, Oxford, and Arizona used small linen samples and carbon-14 dating to establish that the material of the Shroud dated with 95% confidence to 1262 to 1312 AD—about 660 to 720 years old. "These results therefore provide conclusive evidence that the linen of the Shroud of Turin is mediaeval." (P.E. Damon et al. (21 co-authors), *Radiocarbon dating of the Shroud of Turin,* Nature, vol. 337, p. 611 (16 February 1989).)

Living things absorb carbon either from the atmosphere or from eating other living things that absorb carbon from the atmosphere. Atmospheric carbon is 98.9% carbon-12 (that is, made of atoms of carbon with six neutrons joining the six protons) and 1.06% carbon-13 (*seven* neutrons), both of which are stable (non-radioactive). But a small residual amount, between 1 and 1.5 parts per trillion (ppt) at all relevant times (1 ppt being $1/10^{-12}$), is carbon-14 (*eight* neutrons), which is radioactive (emitting a beta ray, converting one neutron to a proton and forming stable nitrogen-14) with a half-life of 5730 years. When a living thing dies, its existing stock of carbon-14 ("radiocarbon") decays at a steady rate, while the carbon-12 and carbon-13 stocks remain constant.

The linen samples were gingerly cut by Professor Giovanni Riggi di Numana under the watchful eye of clerics and scientists. (I wonder if he was nervous.) The three labs were given (1) blind samples of the Shroud, plus three blind control swatches—(2) linen taken from a Nubian excavation of a site known to date from the eleventh to twelfth centuries AD, (3) linen taken from a mummy bearing the name "Cleopatra" in an excavation of a Theban site known to date from 110 BC to 75 AD, and (4) linen from a French clerical cope known to have been made circa 1290-1310 AD.

The labs separately ran all four swatches through their processes, not knowing which was which. A frequently cited methodology for carbon-14 dating is that set forth in Minze Stuiver & Henry A. Polach, *Reporting of ^{14}C Data,* Radiocarbon, vol. 19, no. 3, p. 355 (1977).

The articles do not report the radiocarbon proportions. The ambient proportion does vary over millennia, but it has been rather well calibrated. Here, I assume that the ambient proportion is 1.35 ppt. Based on that assumption and the reported average outcome, I infer that the average of the proportions detected in the Shroud samples is 1.24173 ppt.

The formula for radioactive decay, at rate k, of a substance P over time t from time 0 is:

$$P_t = P_0 e^{-kt}$$

$e^{-kT} = P_t/P_0$, so -kT = ln (P_t/P_0), and if the half-life of radiocarbon is 5730 years, the k factor involves ln 2 or 0.693 and the formula for the age T becomes:

$$T = \frac{\ln(sample,\ ppt/living,\ ppt)}{-0.693} \text{ x } 5730 \text{ years}$$

Here, if the carbon-14 proportions to three significant digits are 1.24 for the Shroud and 1.35 for living material, then the formula is

$$\boldsymbol{T = \frac{\ln(1.24/1.35)}{-0.693} \text{ x } 5730 \text{ years}}$$

That **can be solved using a slide rule!**

First calculate the fraction in the numerator. Using the C and D scales of the slide rule, we find that 1.24 / 1.35 is 0.92. *(Actual value using more significant digits is 0.9198.)*

SLIN 124 D
MUCO 135 CI
RDAC D → 920

So, the equation reduces to

$$T = \frac{\ln 0.92}{-0.693} x \text{ } 5730 \text{ years}$$

Now calculate the numerator, that is, the logarithm. Using the D and LL01 scales of the slide rule, we find that ln 0.92 is −0.0834. *(Actual, −0.08357.)*

MUCO 0.92 LL01
RDAC D → 834

Now calculate the overall fraction. Using the C and D scales of the slide rule, we find that −0.0834 / −0.693 is 0.120. *(Actual, 0.12059.)*

SRIN 834 D
MUCO 693 CI
RDAC D → 120

Finally, calculate the overall product. Using the C and D scales of the slide rule, we find that 0.120 x 5730 is

689 years. *(Actual, 691 years.)*

SLIN 120 D
MUCO 573 C
RDAC D → 689

The average of the Shroud swatch ages reported by the three labs was 691 years before the year 1988, or the year 1297 AD. With one standard deviation of confidence (68%), the range is 1273 to 1288 AD. With two standard deviations of confidence (95%), the range is 1262 to 1312 AD. In any event, the sample is most certainly *not* two thousand years old.

The other swatches were accurately dated within close to one standard deviation of confidence (68%).

- Swatch (2) was dated 937 years, or 1032 to 1159 AD (against a known age of eleventh to twelfth century AD);
- Swatch (3) was dated 1964 years, or 4 BC to 78 AD (against a known age of 110 BC to 75 AD); and
- Swatch (4) was dated 724 years, or 1268 to 1283 AD (against a known age of 1290 to 1310 AD).

Detractors and true believers claim that the Shroud radiocarbon dating is not reliable, or that the linen was contaminated with mediaeval material. The controversy rages on.

Conclusion

The proposed notation system illustrates both the capabilities and the limitations of slide rule calculation. The device yields three or four significant digits, but the user must separately keep track of the units, and on many scales keep track of the decimal place. Applying the system to two historical calculations demonstrates its utility, and allows us to feel the excitement that James Clerk Maxwell must have experienced extracting that final simple square root.